# Entrance Scholarships MATHEMATICS II 

$1^{\text {st }}$ March 2012

Time allowed 1 hour
Show all working.
You may use a calculator

1. Every year the value of a car depreciates by $20 \%$. I buy a car for $£ 20,000$
(a) How much is it worth after 1 year?
(b) How much is it worth after 2 years?
(c) How much is it worth after 5 years?
(d) How many whole years does it take until it is worth less than $£ 1000$ ?
2. 



The diagram shows a circle, centre C, that just fits inside a right-angled, isosceles triangle, PQR . The circle touches the triangle at $\mathrm{L}, \mathrm{M}$ and N . The length $\mathrm{RQ}=2 \mathrm{~cm}$.
(a) Write down the length MQ.
(b) Write down the length LQ
(c) Use Pythagoras's Theorem to show that $\mathrm{PQ}=\sqrt{2} \mathrm{~cm}$
(d) Write down the radius of the circle
(e) Show that the ratio
$($ area of a triangle PQR$):($ area of circle centre C$)=1: \pi(3-2 \sqrt{2})$
3. Solve the following equations for $x$ and $y$
(a) $2^{x} \times 2^{5}=2^{9}$
(b) $\frac{2^{4}}{2^{x}}=2^{-6}$
(c) $2^{x+1} \times 2^{3 x-4}=2^{7-x}$
(d) $\quad 2^{x} \times 2^{y}=2^{9}$

$$
2^{3 x} \times 2^{-y}=2^{7}
$$

(Note: in part (d) there are two equations to be solved simultaneously, so you need to give a value for $x$ and a value for $y$ ).
4.


ABCD is a rectangle in which $\mathrm{AB}=(2 x-2) \mathrm{cm}$ and $\mathrm{BC}=(x+1) \mathrm{cm}$.
(a) If the perimeter of the rectangle is 50 cm , find $x$
(b) If the area of the rectangle is $198 \mathrm{~cm}^{2}$, find $x$
(c) If the length $\mathrm{AC}=10 \mathrm{~cm}$, write down an equation and show that $(5 x+19)(x-5)=0$ and hence find $x$.
5. (a) Copy and complete the following table:

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $x^{3}$ | 8 | 27 |  |  |  |  |  |  | 1000 |
| $x^{3}-1$ | 7 | 26 |  |  |  |  |  |  | 999 |

(b) Copy and complete this table as well:

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $x^{2}+x+1$ | 7 | 13 |  |  |  |  |  |  | 111 |

Use your tables to answer parts (c) - (f)
(c) Calculate $\frac{2^{3}-1}{2^{2}+2+1}$
(d) Calculate $\frac{3^{3}-1}{3^{2}+3+1}$
(e) Calculate $\frac{4^{3}-1}{4^{2}+4+1}$
(f) Calculate $\frac{10^{3}-1}{10^{2}+10+1}$
(g) Calculate $\frac{1000^{3}-1}{1000^{2}+1000+1}$
(h) Write down a formula that summarises all of the above calculations.
(i) Justify your formula.
6. I have three discs, and I want to colour them red, blue or green. These discs are connected by three pieces of string, as shown in the diagram below


All of the discs are to be coloured
(a) In how many different ways is it possible to do this, if those discs that are at either end of a piece of string have to be of different colour?
(b)


In how many ways is this possible if I use only two pieces of string, as shown above?

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